KALMAN FILTER ASSIGNMENT

Adding sensors to a system under the framework of the Kalman filter, can improve the quality of state estimation. This improvement stems from the fundamental principles of filtering and information fusion, where the goal is to optimally estimate the system's state by combining information from multiple measurements and prior knowledge about the system's dynamics and noise characteristics.

# Theoretical Perspective

1. **Information Gain**: Each additional sensor provides new information about the system. Assuming that the sensors are unbiased (i.e., their measurements, on average, reflect the true state of the system), each measurement helps to reduce the uncertainty in the state estimate. This is because, in the Kalman filter framework, the estimation error covariance decreases when more independent measurements are available, assuming that the measurements are not perfectly correlated.

2. **Measurement Noise Reduction**: In the context of the Kalman filter, the measurement update step combines the prior (predicted) state estimate with the new measurement information to produce a posterior (updated) state estimate. The effectiveness of this step is influenced by the measurement noise covariance (R). With multiple sensors of the same type, assuming that the measurement noises are independent, the effective measurement noise covariance can be reduced, leading to a more accurate state estimation.

3. **Resilience and Robustness**: Multiple sensors can provide redundancy, increasing the system's resilience to sensor failures or temporary malfunctions. In scenarios where one sensor gives erroneous data, the other sensors can help mitigate the impact of such outliers on the state estimation process.

# Assumptions

- All sensors have the same noise characteristics (i.e., unbiased and with the same variance).

- Measurement noises between sensors are independent.

- The system and measurement models are linear, and the process and measurement noises are Gaussian, as stipulated by the conditions for applying the Kalman filter.

# Simulation Setup

To illustrate the benefits of using multiple sensors, let's design a simple simulation. Consider a system with a single state variable (for simplicity) where we can control the number of sensors. We will compare the estimation quality using one sensor versus using multiple sensors.

1. **System Dynamics**: Let's assume a very simple dynamic model where \(x\_{t+1} = A x\_t + w\_t\), with \(A = 1\) for simplicity, and \(w\_t\) is normally distributed with mean \(0\) and variance \(Q\).

2. **Measurement Model**: For measurements, \(y\_{t,i} = C x\_t + v\_{t,i}\) for each sensor \(i\), where \(C = 1\). Each sensor has independent measurement noise \(v\_{t,i}\) normally distributed with mean \(0\) and variance \(R\).

3. \*Initial Conditions: We assume \(x\_0\) is normally distributed with mean \(\tilde{x}\_0\) and variance \(P\_0\).

4. \*\*Kalman Filter Implementation\*\*: We will implement a Kalman filter to estimate the state \(x\_t\) from the measurements \(y\_{t,i}\) provided by each sensor. We'll compute the estimation error covariance for cases with 1 sensor and multiple sensors to demonstrate the impact on estimation quality.

### Simulation Execution

Let's conduct two simulations:

1. Using a single sensor.

2. Using \(N\) sensors, where we'll choose \(N = 3\) for illustration.

We'll compare the estimation error covariance in both cases to illustrate the theoretical benefits of using multiple sensors. Let's proceed with setting up the simulation in Python.

A graph showing a line graph

Description automatically generated

The simulation results visually illustrate the state estimation of a dynamic system using different numbers of sensors. The true state of the system is shown alongside the estimated states when using 1 sensor and 3 sensors, respectively.

From the plot, we can observe that the estimation quality improves when using 3 sensors compared to using just 1 sensor. The estimate with 3 sensors tends to follow the true state more closely and exhibits less variance around the true state. This improvement is due to the increased information available from multiple measurements, which helps to better constrain the state estimate and reduce the impact of measurement noise.

### Key Points:

- \*\*Reduced Estimation Error\*\*: The plot demonstrates that using multiple sensors can significantly reduce the estimation error, making the state estimates more accurate and reliable.

- \*\*Information Fusion\*\*: The Kalman filter effectively combines information from multiple sensors, adjusting its estimate based on the variance of the measurement noise and the predicted state variance. This adaptive nature allows it to optimally fuse the available data.

- \*\*Practical Implications\*\*: In practical applications, incorporating multiple sensors can enhance the performance of systems requiring precise state estimation, such as in navigation systems, autonomous vehicles, and tracking applications. However, the benefits must be balanced against the increased complexity and cost associated with additional sensors.

This simulation provides a simplified yet concrete example of how multiple sensors can improve the quality of estimation in dynamical systems, supporting the theoretical advantages of information fusion within the Kalman filter framework.